

Chondrule formation in particle-rich nebular regions at least hundreds of km across

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April 7, 2006

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Chondrules are mm-sized, mostly silicate, spherules that dominate the texture of primitive meteorites¹. Their formation mechanism is debated, but their sheer abundance suggests that the mechanism was both energetic and ubiquitous in the early inner Solar System². The processes suggested, such as shock waves, solar flares, or nebula lightning³⁻⁷, operate on different length scales that have been hard to relate to chondrule properties. Chondrules are depleted in volatile elements, but surprisingly they show little evidence for the associated loss of lighter isotopes one would expect⁸. Here we report a model in which molten chondrules come to equilibrium with the gas that was evaporated from other chondrules, which explains the observations in a natural way. The regions within which the chondrules formed must have been larger than 150-6000 km in radius, and must have had a precursor number density of at least 10 m^{-3} . These constraints probably exclude nebula lightning, and also make formation far from the nebula midplane problematic. The wide range of chondrule compositions may be the result of different combinations of the local concentrations of precursors and the local abundance of water ice or vapor.

The ideal situation of evaporation into vacuum was first described by Rayleigh⁹, and has since been validated by numerous experiments¹⁰. Under these conditions, lighter isotopes in the evaporating (volatile) species have higher thermal velocities, so the residual material becomes preferentially depleted in the lighter isotopes, or isotopically fractionated. While chondrules are depleted in volatile elements like K, Fe, Si, and Mg^{8,11}, the systematic isotopic fractionations associated with evaporation are not seen. There are several hypotheses for why chondrules might show no isotopic fractionations: (1) chondrules came to equilibrium with the gas evaporated from other chondrules, (2) the ambient gas pressure was extremely high, restricting the diffusion of evaporated vapor away from the chondrule, or (3) chondrules were heated and cooled so rapidly that little evaporation occurred. Experiments that simulate chondrule textures tend to preclude case 3 (*ref. 12*). We show that case 2 is implausible, and present a simple physical model for case 1 which specifies the concentrations *and* associated size scales for chondrule formation regions needed to reproduce the observations.

The net evaporation rate for a single chondrule is given by

$$\dot{m}_{evap} = \pi r_c^2 \gamma (\rho_{sat} - \rho_x) v_{th}, \quad (1)$$

where r_c is the chondrule radius, γ is an evaporation coefficient, ρ_{sat} is the saturation vapor mass density, ρ_x is the local vapor density, and v_{th} is the thermal velocity of evaporated atoms or molecules (see online Supplementary Material). If chondrules can reach equilibrium with the evaporated gas, they can avoid preserving the initial isotopic fractionation which accompanies this evaporation^{13,14}. At equilibrium, the ambient vapor density ρ_x must equal the saturation vapor density ρ_{sat} .

We treat two limiting cases leading to this situation¹⁵. Case 1 allows expansion of evaporated vapor into overlapping clouds much larger than the mean chondrule spacing, so that chondrules are bathed in the evaporated material from many other chondrules. Expansion of the evaporated vapor

cloud is limited by diffusion in the ambient hydrogen gas, reaching some characteristic size $R(t)$ on the order of $(\mathcal{D}t)^{1/2}$ in time t , where $\mathcal{D} = 1.4 \times 10^{-4} T^{3/2} / P(\text{bar})$ is the pressure-dependent diffusion coefficient¹⁰.

The local density $\rho_x(t)$ due to overlapping contributions from all evaporating chondrules, at all prior times, can be obtained by summing over all diffusion profiles of all surrounding chondrules as (see online Supplementary Information for details):

$$\rho_x(t) = \int_0^{R_1} 4\pi R^2 n_c dR \int_0^t \frac{\dot{m}_{\text{evap}}(t')}{(4\pi \mathcal{D}t')^{3/2}} e^{-R^2/4\mathcal{D}t'} dt', \quad (2)$$

where R_1 is the radius of the region of chondrule precursors about the local point, having constant volume density n_c , and we use \dot{m}_{evap} from equation (1). If $R_1 > 3\text{-}4\sqrt{\mathcal{D}t}$ (as seen from the point x) the radial integral is known and the equation simplifies to:

$$\rho_x(t) = n_c \pi r_c^2 \gamma v_{th} \int_0^t (\rho_{\text{sat}} - \rho_x(t')) dt', \quad (3)$$

which, for heating time $t = t_h$, has the solution $\rho_x(t) = \rho_{\text{sat}}(1 - e^{-n_c \pi r_c^2 \gamma v_{th} t_h}) = \rho_{\text{sat}}(1 - e^{-a})$, where

$$a = n_c \pi r_c^2 \gamma v_{th} t_h = (A\Phi)(3/4\rho_s r_c)\rho_g \gamma v_{th} t_h. \quad (4)$$

The second equality relates a to the local chondrule-precursor enrichment factor $\Phi = 4\pi\rho_s r_c^3 n_c / 3A\rho_g = 4ar_c\rho_s / 3A\rho_g \gamma v_{th} t_h$ relative to hydrogen by mass, given the silicate cosmic abundance $A \sim 5 \times 10^{-3}$, the chondrule material density ρ_s , and the local gas density ρ_g . How closely ρ_x approaches ρ_{sat} (the value of a) depends not only on the observed isotopic fractionations in the chondrules, but also on the fraction of material in the vapor compared to that in the chondrules¹⁶; values of $a = 3 - 7$ lead to $\rho_x = 0.95 - 0.999\rho_{\text{sat}}$.

In case 2, evaporating material is constrained by very slow diffusion in a high local H_2 (total) gas pressure to a sufficiently small region of radius R_2 , that its vapor density reaches ρ_{sat} in the heating

time t_h ; in this case R_2 can be much smaller than the mean chondrule spacing, and the chondrule merely equilibrates with its own vapor cloud^{17,18}. Pressures required to produce such a situation are on the order of 10 bars (see supplementary material). However, canonical nebula pressures in the asteroid belt region are in the range of 10^{-6} to 10^{-3} bars depending on model assumptions, far too small to produce this effect. We thus consider case 1 to be the most likely option.

The second equality of equation 4 may be used to scale our model to the results of chemical kinetic models¹¹. In **figure 1**, we show that the simple model explains and unifies numerical results over a wide variety of conditions. The first equality of equation (4) can then be solved for n_c for any combination of parameters. Additionally assuming ρ_g , we can solve for Φ . For instance, $a = 6$, $\gamma \sim 0.05$, $r_c = 0.03 - 0.05$ cm, and chondrule heating conditions²⁰ characterized by $T = 1770\text{K}$ for $t_h = 2 \times 10^4$ sec, lead to $n_c = 18 - 7 \text{ m}^{-3}$ respectively. Comparable, or slightly lower, chondrule densities have been derived from other chondrule properties^{3,4}, as discussed in the online Supplementary Material). If the clump size is smaller than $3 - 4\sqrt{\mathcal{D}t_h}$, the values of n_c must be larger.

These conditions characterize the chondrule formation regions. For example, in the context of a shock wave chondrule heating model, both gas and evaporating chondrules have been compressed by about an order of magnitude in the post-shock region^{3,20}. If the pre-shock gas density is a typical minimum mass nebula value $\rho_g \sim 10^{-10} \text{ g cm}^{-3}$, then $\Phi \sim 1400 - 2300$ for $r_c = 0.03 - 0.05$ cm, $a=6$, and $\rho_s = 3.5 \text{ g cm}^{-3}$. Higher pre-shock gas densities^{3,4} of $\rho_g \sim 10^{-9} \text{ g cm}^{-3}$ are found in disk models with mass accretion rates typical of classical T Tauri stars, and lead to smaller values of $\Phi \sim 140 - 230$. Simple “settling to the midplane” is unlikely to satisfy all the relevant criteria, but concentration of the chondrule precursors in nebula turbulence might²¹, as discussed further in the online Supplementary Material.

The above results imply that at least $10^9 - 10^{12}$ chondrules form simultaneously in a volume of radius $R_1 \geq 3 - 4\sqrt{\mathcal{D}t_h} \sim 0.5 - 20\text{km}$ (assuming $T = 1770\text{K}$, $t_h = 2 \times 10^4$ sec, and local gas pressure of $10^{-3} - 10^{-5}$ bars). Chondrules in this co-formed batch equilibrate chemically with each other within t_h , through rapid exchange of their evaporated atoms. If more than 99% of all chondrules in a region of radius R are to escape significant isotopic fractionation, then $(1 - R_1/R)^3 > 0.99$; thus $R > 300R_1 \approx 150\text{-}6000$ km (or larger values of n_c are needed).

This lengthscale distinguishes between competing models of chondrule formation. Nebula lighting, for instance, affects a region no more than a few km across⁷. Bow shocks from planetesimals are comparable in size to the planetesimal radius, and thus would also be much smaller than our inferred lengthscales, except in the unlikely event that most planetesimal growth had already been accomplished^{4,22}. If chondrule formation regions at least 150-6000 km in scale are needed to satisfy the isotopic fractionation constraints, these smaller-scale mechanisms can be excluded independently of the usual arguments based on heating and cooling timescales, which are subject to different uncertainties. Nebula shocks due to gravitational instabilities or jovian density waves are on far larger scales, comparable to the nebula thickness^{3,20}; here the effective spatial scale of the chondrule forming event is provided by the spatial scale of the dense cloud of precursors, which is amenable to modeling²¹. However, fast shocks from solar flares⁶ act primarily at low ambient nebula densities $\sim 5 \times 10^{-12}$ to 5×10^{-11} g cm⁻³, requiring chondrule precursor concentration factors to be 20 – 200 times larger, *and* over volumes $> 10 - 30$ times larger in radius (because of the dependence of \mathcal{D} on density) than for $\rho_g \sim 10^{-9}$ g cm⁻³.

Finally, we note that these results have broader implications. A long standing concern with nebula melting scenarios has been that chondrule melts are not stable given cosmic abundances; however, the conditions we derive here to preclude isotopic fractionation also stabilize silicate

melts^{11,14,16}. Moreover, the compositions of these melts will reflect local conditions. For instance it is well known that most chondrules in ordinary and carbonaceous chondrites are too oxidized to have formed in a canonical solar mixture¹⁶, requiring local oxygen abundances that are orders of magnitude larger than nominal. Such oxygen enrichments would occur if $\Phi \gg 1$ and the chondrule precursor materials were FeO-rich^{11,16}. Addition of H_2O can also affect oxidation state; this could occur either if chondrule precursors contained water ice, or if chondrules formed sunward of water ice evaporation fronts^{23,24} where water was enriched in the vapor. Water is likely to carry large oxygen isotopic anomalies produced by UV photodissociation in the outer nebula or protostellar cloud^{25–27}. Consequently, local variations in the chondrule precursor concentration factor would produce a range of possibly correlated elemental, isotopic and redox properties even amongst chondrule batches formed in the same event. More extensive observations could constrain the small fraction of chondrules which *do* show isotopic fractionations. The wide range of chondrule compositions, an important piece of the chondrule puzzle, may at least be partially caused by different combinations of the local concentration of chondrule precursors and global effects such as evaporation fronts. Finally, while beyond the scope of this paper, the model outlined above could also be used to constrain the formation conditions of the enigmatic calcium-aluminium-rich inclusions (CAIs) that *do* preserve variable degrees of isotopic fractionation^{8,10}.

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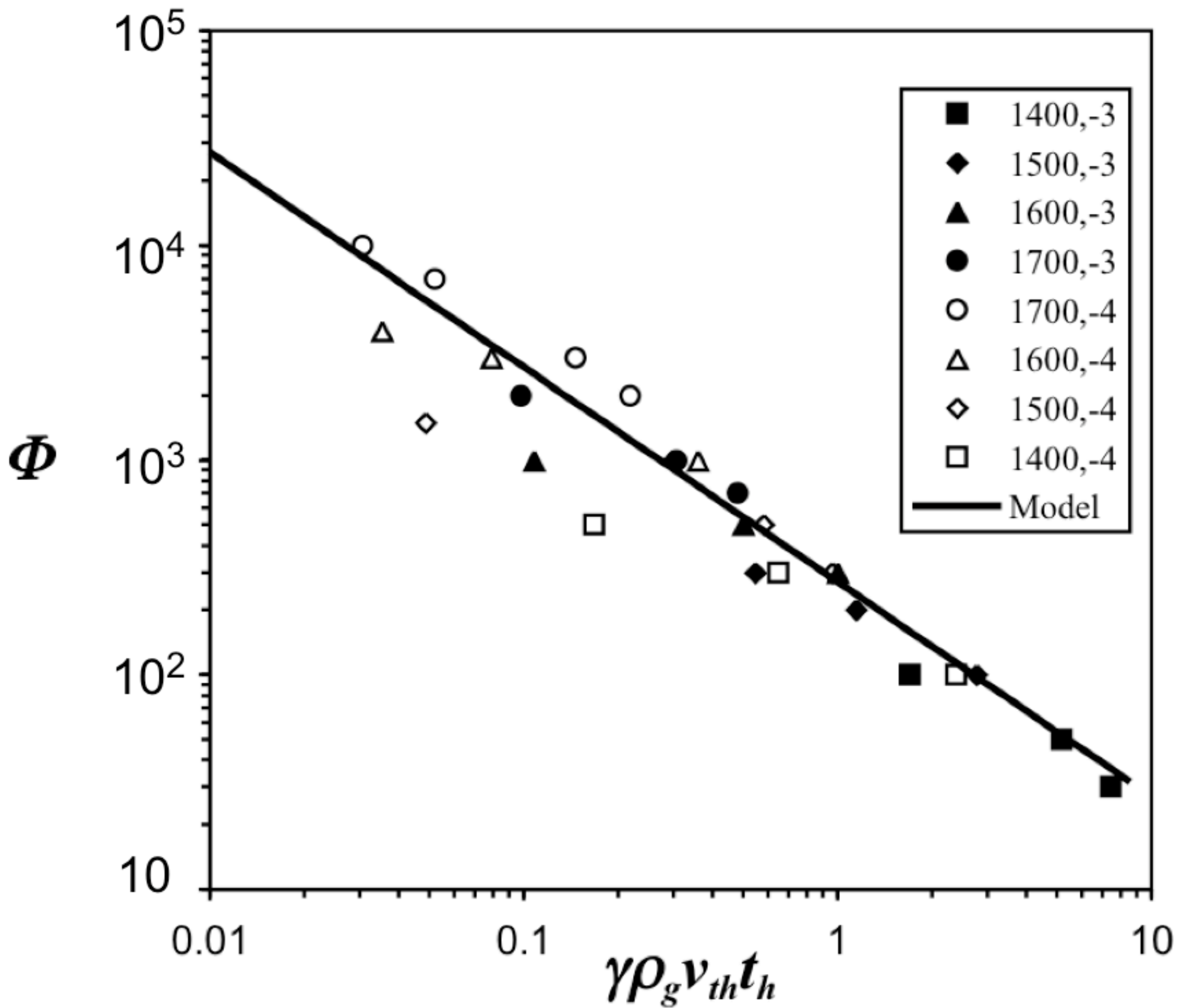
Supplementary information is linked to the online version of the paper at www.nature.com/nature.

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Acknowledgments: We are very grateful to an anonymous referee whose comments led us to develop the Green’s function solution presented in the online Supplementary Material which is both more rigorous and more straightforward than our prior method, and whose comments helped us clarify the paper in other ways as well. We thank Ed Young for several helpful discussions and Alan Boss, Larry Nittler, and Fred Ciesla for comments which improved the manuscript. This work was supported by NASA’s Planetary Geology and Geophysics, Origins of Solar Systems, and Cosmochemistry Programs.

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Figure legend: The mass density in clumps where chondrules form is captured in our model by a simple combination of local properties. In this figure, the mass enrichment factor Φ over cosmic abundance (equation 4) required to forestall Mg isotopic fractionation (≤ 0.25 per mil per amu), as a function of the combined variable $\rho_g \gamma v_{th} t_h$. The line is the second equality of equation 4, and the points are results from numerical models of chemical kinetics¹¹. Our simple model captures the form of the dependence of Φ on the combined local quantity $\rho_g \gamma v_{th} t_h$ very well, with numerical results for pressures of both 10^{-3} and 10^{-4} bars collapsing into the same trend; we require $a = 6 \pm 1$ for a quantitative match. Refinements beyond the scope of this paper greatly improve the fit, as discussed more in the online Supplementary Material. An alternate, but less often used, approach to calculating chondrule evaporation in non-equilibrium situations leads to an equally good fit, but the values of γ are roughly three times as large for this model which would decrease n_c accordingly. Filled symbols: nebula pressure of 10^{-3} bars; open symbols: nebula pressure of 10^{-4} bars. Temperatures vary between 1400C and 1700C.



1 Supplementary Discussion

General relationships and assumptions: Regarding expressions for evaporative flux, equation (1) is identical to alternate forms which involve saturation pressure P_{sat} rather than density: $\dot{m}_{evap} = 4\pi r_c^2 \gamma (P_{sat} - P_x) \sqrt{\mu/2\pi \mathcal{R}T}$ g cm⁻²sec⁻¹, where μ is the molar mass of the evaporated species and $\mathcal{R} = 8.3 \times 10^7$ erg deg⁻¹ mole⁻¹ is the universal gas constant^{2,3}, since, in general, $P = \rho \mathcal{R}T/\mu$ and $v_{th} = \sqrt{8\mathcal{R}T/\pi\mu}$. For instance, for Mg, $\mu = 24$ g/mole. For $T = 1770$ K, this leads to $v_{th} = 1.2$ km/s. Notice this is the full thermal speed, larger than the 1D velocity or the nebula sound speed.

Solving for the local density in a situation where clouds from many chondrules overlap: Regarding diffusion of a tracer species, the radial distribution of mass density (g/cm³) into three dimensional distance R , as a function of time t subsequent to a point release of some injected mass $dM = \dot{m}_{evap} dt$, is given by:

$$\rho(R, t) = \frac{dM}{(4\pi \mathcal{D}t)^{3/2}} e^{-R^2/4\mathcal{D}t},$$

which is the Green's function from an impulsive point mass release. The local density at time t due to overlapping contributions from all evaporating chondrules, at all prior times, can then be written as

$$\rho_x(t) = \int_0^{R_1} 4\pi R^2 n_c dR \int_0^t \frac{\dot{m}_{evap}}{(4\pi \mathcal{D}t')^{3/2}} e^{-R^2/4\mathcal{D}t'} dt',$$

where R_1 is the radius of the region of chondrule precursors, wherein n_c is assumed constant. It is easily shown that the mass fraction contained within some radius R_1 is about 80% for $R = 3\sqrt{\mathcal{D}t}$ and about 95% for $R = 4\sqrt{\mathcal{D}t}$. For clump sizes R_1 larger than these values (as seen from the point x) we can simply integrate to infinity. We substitute for \dot{m}_{evap} from equation (1), and exchange the order of integration, to obtain

$$\rho_x(t) = 4\pi^2 n_c r_c^2 \gamma v_{th} \int_0^t \frac{(\rho_{sat} - \rho_x(t')) dt'}{(4\pi \mathcal{D}t')^{3/2}} \int_0^\infty R^2 e^{-R^2/4\mathcal{D}t'} dR.$$

The integral over R has the value $(4\mathcal{D}t')^{3/2}\sqrt{\pi}/4$ (assuming $R_1 > 3 - 4\sqrt{\mathcal{D}t}$). Thus we obtain

$$\rho_x(t) = n_c \pi r_c^2 \gamma v_{th} \int_0^t (\rho_{sat} - \rho_x(t')) dt'.$$

Differentiating both sides with respect to t leads to

$$\frac{d\rho_x(t)}{dt} = n_c \pi r_c^2 \gamma v_{th} (\rho_{sat} - \rho_x(t)),$$

and thus

$$\frac{d\rho_x(t)}{(\rho_{sat} - \rho_x(t))} = n_c \pi r_c^2 \gamma v_{th} dt$$

leading directly to

$$\rho_x(t) = \rho_{sat}(1 - e^{-n_c \pi r_c^2 \gamma v_{th} t}) = \rho_{sat}(1 - e^{-a}).$$

Overall, our development assumes r_c is constant, which is reasonable when most of the silicate mass is in the chondrules rather than in the vapor¹⁶; also, a constant average temperature (through v_{th}) is assumed over the duration t_h . This expression for ρ_x is identical to a mathematical formulation in which n_c chondrules per unit volume evaporate at fixed temperature T in a closed box¹¹. In such a system the pressure P_x changes at a rate

$$dP_x/dt = \frac{\mathcal{R}T n_c 4\pi r_c^2}{\sqrt{2\pi\mathcal{R}T\mu}} (P_{sat} - P_x) = b(P_{sat} - P_x),$$

where $P = \rho\mathcal{R}T/\mu$. Integrating this equation leads directly to $P(t) = P_{sat}(1 - e^{-bt})$. Substituting $v_{th} = \sqrt{8\mathcal{R}T/\pi\mu}$ shows that $bt = a$ above.

The value of a is determined by how closely the local gas vapor density has to approach ρ_{sat} , which is in turn determined by the desired upper limit on isotopic fractionation. Mass balance implies that, if the amount of some element in the vapor phase is a small fraction X_g of the total, the melt can retain low isotopic fractionations δ_{ch} even while the gas displays much larger fractionations δ_g . Specifically, $\delta_g/\delta_{ch} = (1 - X_g)/X_g$.

One can estimate how long an atom survives before it encounters a chondrule under the conditions derived above, as a measure of how well the gas interacts with the chondrule cloud. The encounter time is given by $t_{enc} = 1/(n_c \pi r_c^2 v_{th}) = \gamma t_h / a$ sec; the chemical equilibration time is longer by a factor γ^{-1} , and thus equal to t_h / a .

Diffusion of vapor in ambient hydrogen gas: Diffusion depends on the density of the ambient gas and the thermal speed of the diffusing material. The diffusion coefficient for Mg or SiO in hydrogen is given by *ref* (10), equation 16, as $\mathcal{D}(\text{cm}^2\text{sec}^{-1}) \approx 1.4 \times 10^{-4} T^{3/2} / P$, where T is in degrees K and P is the total (H_2) pressure in bars. In this regime, a cloud will diffuse in time t_h to size $R = (\mathcal{D} t_h)^{1/2} = (2 \times 10^5 / P(\text{bars}))^{1/2} \sim 460 \text{ cm } (P(\text{bars}))^{-1/2}$ for $t_h = 2 \times 10^4$ sec and $T = 1770\text{K}$. For canonical nebula pressures of 10^{-6} to 10^{-3} bars, vapor diffuses to distances on the order of $R_1 \sim 0.15 - 5 \text{ km}$ in time t_h . Note that a transcription error led to incorrect numerical results in *ref* 18 (E. Young, personal communication, 2006).

High ambient pressures required to preclude isotopic fractionation: In the pressure-controlled regime of case 2, the vapor density is approximated by the total mass evaporated in the heating time t_h divided by the volume (neglecting the complication of the changing evaporation rate):

$$\rho_x = 3\dot{m}_{evap} t_h / 4\pi R_2^3 = 3\gamma \rho_{sat} v_{th} \pi r_c^2 t_h / 4\pi R_2^3. \quad (1)$$

Setting $\rho_x = \rho_{sat}$ explicitly gives $3r_c^2 v_{th} \gamma t_h / 4R_2^3 = 1$. Solving for R_2 we get about 40-60 cm, which turns out to be comparable to the required chondrule spacing $n_c^{-1/3}$ of case 1. That is, if the radius of an individual chondrule's evaporation cloud can be restricted to less than 40-60 cm by high total gas pressure, then the requirement $\dot{m}_{cond} = \dot{m}_{evap}$ is satisfied regardless of the chondrule spacing¹⁸. However, using the diffusion coefficient from *ref.* 10, we find that the local gas pressure can restrict evaporation clouds in this way only if it exceeds 20 bars. A more sophisticated treatment¹⁰ solves a

tracer diffusion equation for the vapor pressure of the evaporating material at the particle surface in the presence of this limited diffusion. The condition for preventing isotopic fractionation can be simply derived from equations (20) and (21) of Richter et al. as $\gamma r_c v_{th}/4\mathcal{D} \gg 1$. This condition is seen graphically as the high-pressure asymptote in figure 13 of *ref. (10)*, and is closely related to our equation (5) above which can be written as $\gamma r_c v_{th}/4\mathcal{D} = R_2/3r_c \gg 1$. A similar criterion can be obtained using the Peclet or evaporation number, which replaces the imposed heating timescale t_h by the time needed to evaporate all the species in question from the chondrule, (*eg. ref. (10)* p. 354) but seems less appropriate in the case at hand. Using the above expression for \mathcal{D} , with $r_c=0.03$ cm, $v_{th} = 1$ km/s at $T = 1770$ K, and $\gamma = 0.05$, one obtains $P \gg 0.4$ bar. The requirements can be refined using figure 13 of *ref. (10)* which, while not so stated, is appropriate for a particle size of approximately 0.3 cm; thus for chondrule-sized particles ($r_c = 0.03$ cm) the entire curve must be shifted to the right by a factor of ten, implying that the asymptotic $P \sim 10$ bars, in agreement with our simple estimate above.

Details of the chemical kinetic models presented in figure 1: Figure 1 shows how expressing the parameters of a series of chemical kinetic calculations in combinations predicted by our simple model causes them all to collapse onto a simple trendline. The fit of figure 1 was achieved using equation (4), by calculating the product $\gamma \rho_g v_{th} t_h$ for several runs from *ref. (11)* that achieved chondrule Mg isotopic fractionation of 0.25 per mil per amu. The numerical experiments used temperature- and composition-dependent γ values adjusted from vacuum measurements to fit the experiments in H_2 of *ref. (10)*; thus, each symbol has a particular γ applied, ranging from $\gamma = 0.037(1700\text{C})$ to $\gamma = 0.067(1400\text{C})$ for final compositions with $\text{FeO} < 1$ mole%, and $\gamma=0.039(1700\text{C})$ to $\gamma=0.036(1400\text{C})$ for final compositions with $\text{FeO} > 1$ mole%.

Other parameters used were the initial chondrule radius $r_c = 0.47\text{mm}$ from *ref. (11)* and solid

density $\rho_s=3.5 \text{ g cm}^{-3}$. Calculation of the line in figure 1 then has only the single free parameter a . For convenience we simply adopt $\gamma = 0.05$ and $T = 1500C$ in applications of the model to other situations. Matching the numerical models of Alexander¹¹ requires that $a \approx 6 \pm 1$ in our simple model for chondrule Mg to approach the low observed levels. However, there are several effects in the numerical simulations that are beyond the scope of this paper, which, if taken into account, would decrease a . For instance, most of the Mg is likely to be in the melt during chondrule formation (see “Solving for the local density..”, above). A more careful treatment of this and several other small effects which bias the current chemical kinetic model results greatly decreases the scatter of points about our simple model line (work in preparation).

All points shown were calculated using the standard EQR model, but Alexander¹¹ (see also *ref. 28*) notes potential problems with the EQR model for situations in which chondrules are not in chemical and thermal equilibrium (*eg.* shock wave heating), and proposed a new model (the PCR model), which follows individual gas-chondrule and gas-phase reactions explicitly but is less widely used. His solutions for Φ as functions of $\gamma\rho_g v_{th} t_h$ from the PCR model are systematically lower by about a factor of three, attributable to smaller γ values preferred by this model, so inferred n_c and Φ values would thus decrease by that factor. In this paper we rely primarily on the EQR results because this model is more widely used.

Analysis of other meteoritic properties leads to comparable chondrule number densities as we obtain here. Analyses of the abundances of compound chondrules^{3,4} were recently summarized²⁹ as requiring $n_c \sim f_{comp}/(4\pi\sqrt{2}r_c^2\overline{V}t_{coll})$, where f_{comp} is the fraction of compound chondrules, \overline{V} is some relative velocity, and t_{coll} is some duration. The pre-shock relative velocities in turbulence for such small particles are probably on the order of 10 cm/sec ³⁰ and the plastic time is on the order of 10^4 sec ²⁹. Relative velocities might be stirred by the shock itself, based on the

likely variance in particle size, to perhaps 10^4 cm/sec, but these high velocities are damped by gas drag in times as small as 10^2 sec²⁹. This leads to a range of $n_c \sim 0.1\text{-}30$ m⁻³. On entirely different grounds, *refs 4 and 18* find that chondrule cooling rates depend on the density of chondrules, and use current estimates of cooling rates to obtain a preferred range of $n_c \sim 0.5\text{-}5$ m⁻³.

How are these high chondrule enrichments produced? Chondrule-sized particles simply cannot “settle into the midplane” to produce concentrations as high as required unless the nebula is entirely non-turbulent^{31,32}. In this case particles of all sizes, not just chondrules, would be melted and intermingled³³, whereas chondrules are extremely well size-sorted^{19,21} in each meteorite, and often covered by fine-grained dust rims that accreted long after they cooled and well before they accumulated into a meteorite parent body. It has been found that nebulae with canonical turbulent intensities *selectively concentrate* chondrule-sized particles with size distributions which are indistinguishable from those observed²¹ into dense zones where concentration factors (enrichment factors) of order 100 are common; even larger values of concentration are feasible. This is known as turbulent concentration.

If only some fraction f_c of available silicates - particles having chondrule mass m_c - are concentrated by a factor C over solar, then we can solve for the required C based on the values of Φ determined from the observations as $C = \Phi/f_c = 4a\rho_s r_c / 3A\rho_g f_c \gamma v_{th} t_h$. Values of $\Phi \sim 140 - 230$ and $f_c = 0.5$ lead to $C \sim 280 - 460$, respectively, for our adopted parameters. It remains to be shown quantitatively whether this mechanism can achieve adequate concentration C on the appropriate lengthscales R_1 with reasonable probabilities to account for isotopic fractionation being so commonly lacking in chondrules. For instance, current models indicate that only 5 - 30% of preferentially concentrated particles lie in regions with $C > 200$, whereas 30 - 70% of preferentially concentrated particles lie in regions with $C > 30$ (*ref 21*). Thus, concentration factors we derive in

this paper of 300-500 seem on the high side for *most* chondrules (or precursors) to reside in, based on these models. However, factors of order unity are important for these probability distributions, so improvements in the model and chondrule properties involving factors of order unity will thus be important (*i.e.*, in ρ_s and γ which affect n_c and a , as discussed above). Testing of the turbulent concentration mechanism, and other models, will be possible using the improved estimates of the spatial scales and concentration factors that the simple model presented here produces. It is surely of interest that turbulent concentration appears to have the potential to explain several independent chondrule and chondrite properties at once.

References for Supplementary Material

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